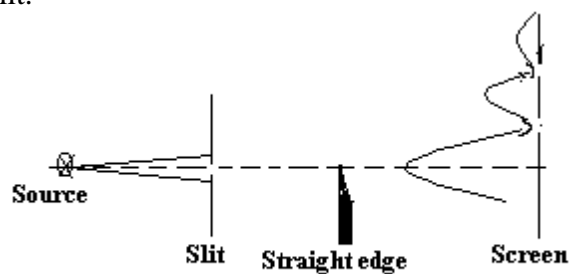


DIFFRACTION

According to corpuscular theory of light, light travels in straight lines. It is observed from various experiments that light deviates from straight line path when it passes over the edges of opaque objects or slits. It is found that light bends into the region of the geometrical shadow of the object. This deviation of light is very small when the dimensions of the slit or size of the object are very large compared to the wavelength of the light waves. When the width of the slit is comparable to the wavelength, the deviation of light is more and the waves spread over all the surface behind the slit.



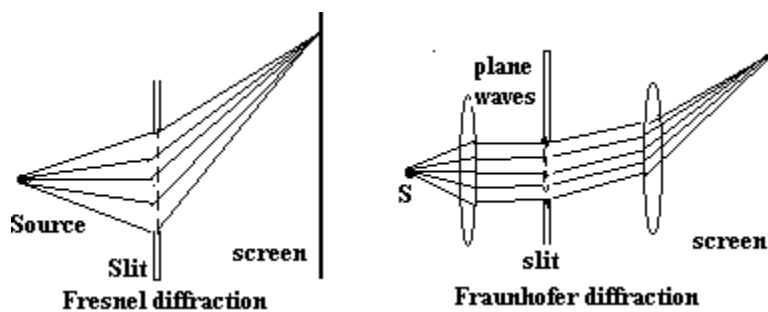
Diffraction due to a straight edge

X is a source of monochromatic light and light waves diverging from the slit S pass an obstacle with straight sharp edge AB parallel to the slit. The screen is placed parallel to the straight edge. According to the corpuscular theory, light travels in straight lines. So complete uniform illumination should be obtained above the point O and complete darkness below O, in the geometrical shadow. But the experiments showed that above O, unequally spaced alternate bright and dark bands of varying intensity up to a certain region beyond which there is uniform illumination. Below on the screen, in the geometrical shadow there is faint illumination whose intensity falls continuously to zero. There are no diffraction fringes inside the shadow.

Thus when light falls on objects or small apertures whose size is comparable with the wavelength of light, the light deviates from the straight line propagation and bends round the corners of the obstacle or apertures and enters in the geometrical shadow. **The bending of light around the edges of the object and entering light into the geometrical shadow of the object when it passes over the edges of the object is called 'diffraction'. It is found that diffraction produces bright and dark fringes known as diffraction bands or fringes.**

According to Fresnel, the diffraction phenomenon is due to mutual interference of the secondary wavelets originating from various points of the wavefront which are not blocked by the obstacle. Fresnel applied Huygen's principle of secondary wavelets in conjunction with the principle of interference and calculated the position of fringes. It was observed that the calculated results were in good agreement with the observed diffraction. It should be remembered that the diffraction effects are observed only when a portion of wavefront is cutoff by some obstacle.

Types of diffraction: In the case of light waves, the diffraction phenomenon at the edges of the obstacles or at the slits depends on the nature of the incident wavefront. There are two types of diffraction known as Fresnel diffraction and Fraunhofer diffraction



Fresnel diffraction: In this type of diffraction, the source of light and screen are at finite distance from the obstacle or aperture. The incident wavefront is either spherical or cylindrical. As a result the phase of the secondary wavelets is not the same at all points in the plane of the aperture. No lenses are used to make the rays parallel or convergent

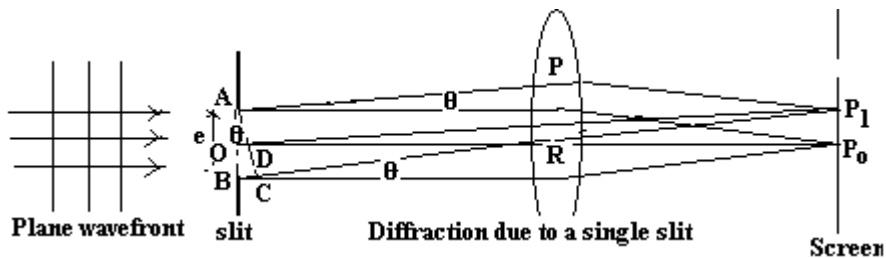
Fraunhofer diffraction: In this class of diffraction, the source of light and the screen are at infinite distances from the object. This may be achieved by using two convex lenses. The incident wavefront is plane. As a result, the secondary wavelets are in the same phase at every point in the plane of the aperture.

Fraunhofer diffraction conditions are established in the lab by using two convergent lenses. The first lens converges the diverging waves from the source into a plane wave. The second lens causes the plane waves leaving the diffracting aperture to converge to a point aperture to converge to a point

Differences between interference and diffraction phenomena:

	INTERFERENCE	DIFFRACTION
1	Interference is the result of interaction of light coming from two different wavefronts (originating from two coherent sources)	Diffraction is the result of interaction of light between the secondary wavelets originating from different points of the exposed parts of the same wavefront
2	Interference fringes may or may not be of same width	The width of diffraction fringes are never Equal
3	Points of minimum intensity are perfectly dark	Points of minimum intensity are not perfectly dark
4	All bright bands are of uniform intensity	All bright bands are not of same intensity

Fraunhofer diffraction due to a single slit:



AB is a narrow slit of width e perpendicular to the plane of the paper. To obtain a Fraunhofer diffraction pattern, the incident light is collected on the screen which is placed in the focal plane of a convex lens. The source of light should be either at a large distance from the slit or a collimator lens must be used. The slit is illuminated by monochromatic light of wavelength λ propagating normally to the slit. According to Huygen's theory, every point of the wavefront in the plane of the slit is a source of secondary wavelets which spread out to the right in all directions. The secondary wavelets traveling normally to the slit, i.e. along the direction OP_0 are brought to the focus at P_0 by the lens. Thus P_0 is a bright central fringe. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the wavefront travel the same distance in reaching P_0 and hence the path difference is zero. The secondary waves reinforce one another and P_0 will be a point of maximum intensity.

The secondary wavelets traveling at an angle θ with the normal are focused at a point P_1 on the screen. The point P_1 will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Ray r_1 originates at the top of the slit and ray r_2 at its center. If θ is chosen such that OD is equal to $\lambda/2$, then r_1 and r_2 will be out of phase and P_1 is a dark point. In fact every ray from upper half of the slit will be canceled by a ray from lower half of the slit, originating at a point $(e/2)$ below the first ray. **Path difference between r_1 and r_2 is $(e/2) \sin \theta$** The point P_1 , the first minimum of the diffraction pattern,

$$(e/2) \sin \theta = \lambda/2 \quad \text{or} \quad e \sin \theta = \lambda$$

Now the slit is divided into four equal zones, with a ray leaving at the top of each zone. Let θ be chosen so that the path difference between rays r_1 and r_2 is $(e/4) \sin \theta = \lambda/2$ and the rays r_1 and r_2 cancel at P_2 . Rays r_3 and r_4 that originate at the top of third and fourth quarters of the slit will also be half wavelength out of phase and will also cancel. This is the second point of zero intensity $(e/4) \sin \theta = \lambda/2$ or $e \sin \theta = 2\lambda$

By extension, the general formula for minimum in the diffraction pattern is

$$e \sin \theta = m\lambda \quad \text{where} \quad m = 1, 2, 3, \dots$$

there is a maximum approximately half way between each adjacent pair of minima

The intensity at point P_1 on the screen depends on the relative phases of light received from various parts of the slit

Let us consider that the width of the slit is divided into n equal parts and the amplitude of the wave from each part is 'a' (because the width of each part is same). The phase difference between any two consecutive waves from these parts would be

$$(1/n) (\text{total phase}) = (1/n) (2\pi/\lambda) e \sin \theta = d$$

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = a \sin (nd/2) / \sin (d/2) = a \sin [n (2\pi/n\lambda) (e \sin \theta) / 2] / \sin [(2\pi/n\lambda) (e \sin \theta) / 2]$$

$$= a \sin [(\pi e \sin \theta) / \lambda] / \sin [(\pi e \sin \theta) / n\lambda]$$

$$= a \sin \alpha / \sin (\alpha/n) \quad \text{where} \quad \alpha = (\pi e \sin \theta) / \lambda$$

$$= (a \sin \alpha) / (\alpha/n) \quad \text{since} \quad \alpha/n \text{ is very small}$$

$$R = (na \sin \alpha) / \alpha = A \sin \alpha / \alpha$$

When $n \rightarrow \infty$, $a \rightarrow 0$ but the product $na = A$ remains finite
 Now the intensity is given by $I = R^2 = A^2 (\sin \alpha / \alpha)^2$

Principal maximum ::

The expression for resultant R can be written in ascending powers of α as

$$R = (A/\alpha) (\alpha - \alpha^3/3! + \alpha^5/5! - \alpha^7/7! + \dots)$$

$$= A (1 - \alpha^2/3! + \alpha^4/5! - \alpha^6/7! + \dots)$$

If the negative terms vanish, the value of R will be maximum i.e. $\alpha = 0$

Therefore $\alpha = (\pi e \sin \theta)/\lambda = 0$ or $\sin \theta = 0$ or $\theta = 0$

Now the maximum value of R is A and the intensity is proportional to A^2 . The condition $\theta = 0$ means that this maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is known as **principal maximum**

Minimum intensity positions : The intensity will be minimum when $\sin \alpha = 0$. The values of α which satisfy this equation are

$$\alpha = \pm \pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \dots = \pm m\pi$$

or $(\pi e \sin \theta)/\lambda = \pm m\pi$

$$e \sin \theta = \pm m\lambda \quad \text{where } m = 1, 2, 3, \dots$$

In this way we obtain the points of minimum intensity on either side of the principal maximum. The value of $m = 0$ is not admissible because for this value, $\theta = 0$ and this corresponds to principal maximum.

Secondary maxima : In addition to principal maximum at $\alpha = 0$, there are weak secondary maxima between equally spaced minima. The positions can be obtained with the rule of finding maxima and minima of a given function in calculus. Differentiating the expression of I with respect to α and equating to zero, we have

$$dI/d\alpha = d/d\alpha [A^2 (\sin \alpha / \alpha)^2] = 0$$

$$\text{or } A^2 (2 \sin \alpha / \alpha) (a \cos \alpha - \sin \alpha) / \alpha^2 = 0$$

$$\text{either } \sin \alpha = 0 \quad \text{or } (a \cos \alpha - \sin \alpha) = 0$$

The equation $\sin \alpha = 0$ gives the values of α (except 0) for which the intensity is zero on the screen. Hence the positions of maxima are given by the roots of the equation

$$(a \cos \alpha - \sin \alpha) = 0 \quad \text{or} \quad \alpha = \tan \alpha$$

The value of α satisfying the above equation are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan \alpha$ on the same graph. The points of intersection of two curves give the values of α which satisfy the equation $\alpha = \tan \alpha$. The plots of $y = \alpha$ and $y = \tan \alpha$ are shown in Fig

The points of intersection are $\alpha = 0, \pm 3\pi/2, \pm 5\pi/2, \dots$

or more exactly to $\alpha = 0, \pm 1.43\pi, \pm 2.462\pi, \pm 3.471\pi, \dots$

$\alpha = 0$, gives the principal maximum

Substituting approximate values of α in equation $I = R^2 = A^2 (\sin \alpha / \alpha)^2$, we get the intensities in various maxima

$$I = A^2 \quad (\text{principal maxima})$$

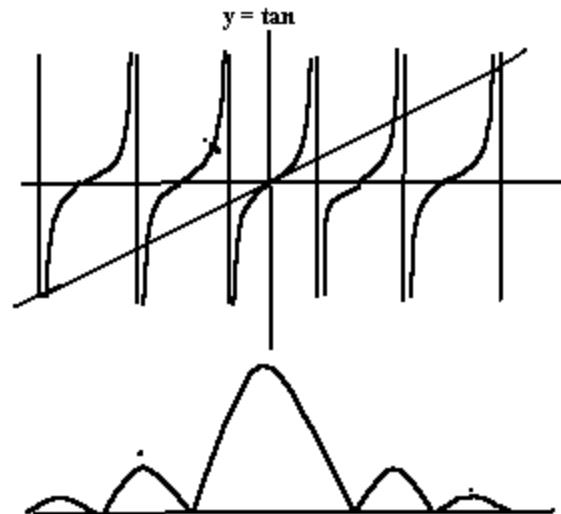
$$I_1 = A^2 (\sin 3\pi/2 / 3\pi/2)^2 = A^2 / 22 \quad \text{approximately}$$

$$I_2 = A^2 (\sin 5\pi/2 / 5\pi/2)^2 = A^2 / 62 \quad \text{approximately}$$

From these expressions of I_0, I_1, I_2, \dots it is evident that most of the incident light is concentrated in the principal maximum

Intensity distribution graph :

A graph showing the variation of intensity with α is shown here. **The diffraction pattern consists of a central maximum occurring in the direction of incident rays. There are subsidiary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm 3\pi/2, \pm 5\pi/2, \dots$. Between subsidiary maxima, there are minima at positions $\alpha = \pm \pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \dots$**



Summary :

1. When a wave encounters an obstacle or an slit or an aperture with a size comparable with the wavelength of the waves, those waves spread out as they travel and ,as a result, undergo interference. This is called diffraction
2. When a wave encounters an obstacle secondary waves are generated. The interference of secondary waves produces a diffraction pattern.
3. The Fraunhofer diffraction pattern produced by a single slit consists of a central bright fringe flanked by alternating bright and dark fringes of much lower intensities
4. Waves passing through a long narrow slit of width e produce a single slit diffraction pattern that includes a central maximum and other maxima separated by minima located at angles θ to the central axis that satisfy

$$e \sin \theta = m\lambda \quad \text{for } m= 1,2,3, \dots$$

The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2, \quad \text{where } \alpha = (\pi e \sin \theta) / \lambda$$

And I_m is the intensity at the center of the pattern

University Questions.

- 1) **Distinguish between Fresnel and Fraunhofer diffraction**
- 2) **Explain the nature of fringes obtained in the case of diffraction due a single slit**
- 3) **Write differences between diffraction and interference phenomena**
- 4) **Discuss the quantitative description of diffraction at a single slit**

POLARISATION

Introduction:

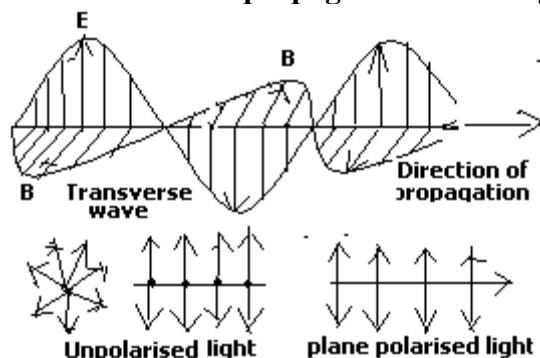
The experiments on interference and diffraction have established that light is a form of wave motion. But these phenomena do not reveal the character of this wave motion i.e., whether it is longitudinal or transverse. In longitudinal waves, the particles of the medium move to and fro in the direction of propagation of the wave as in the case of sound waves. In transverse waves the particles of the medium vibrate up and down right angles to the direction of propagation of the wave.

Maxwell's electromagnetic theory predicts light to be a transverse wave in which the fluctuating electric and magnetic fields are perpendicular to each other and to the direction of propagation. The studies on polarization of light waves experimentally established this transverse nature of light. Longitudinal waves like sound waves show interference and diffraction effects but do not exhibit polarization.

An experimental basis for believing light to be a transverse wave was provided by Thomas Young in 1817. He allowed a light beam to fall on a calcite crystal to produce two separate beams. Surprisingly, these beams, although coherent, produced no interference fringes but only uniform illumination. Young deduced from this that light must be a transverse wave and that the plane of vibration in the two beams must be at right angles to each other. Two wave disturbances that act at right angles to each other cannot show interference effects.

Unpolarised light:

The light propagated in a given direction consists of independent wave trains whose planes of vibration are randomly oriented about the direction of propagation. Such light, though still transverse, is unpolarised. **The electric field vectors vibrate in all directions perpendicular to the direction of propagation. The light is transverse but there is no preference direction of vibration of the electric field vector. The random orientation of the planes of vibrating electric vectors leads to symmetrical distribution about the direction of propagation.** Such a light is called unpolarised light



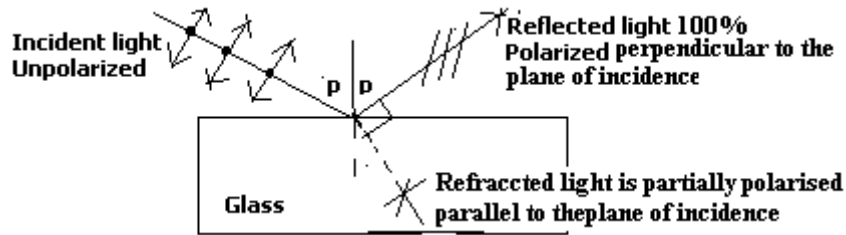
Polarised light:

Light wave which has acquired the property of one-sidedness or a light wave that is unsymmetrical about the direction of propagation is called polarized light. **If in a transverse wave, the direction of oscillation of electric vector at all points are strictly confined to the same single plane, the wave is said to be plane polarized or linearly polarised . This means that the vibrations of the electric field vector are parallel to each other for all points on the wave.** At any such point the vibrating **E** vector and the direction of propagation form a plane, called the *plane of vibration*. In such a plane polarized wave all such planes are parallel. We always define the direction of polarization of the EM wave as the direction of the electric field vector **E**

Polarisation by reflection:

The simplest way of producing plane polarized light is by reflection. When light is reflected from the surface of a transparent medium like glass, it becomes partially polarized. The degree of polarization

varies with the angle of incidence. At a certain angle of incidence called the ‘**polarizing angle**’ (angle of polarization), the reflected light is completely polarized.



Brewster proved that the tangent of the angle of polarisation (p) is numerically equal to the refractive index of the medium.

$$\mu = \tan p \quad (\text{Brewster's law})$$

A direct deduction from this law is that when light is incident at the polarizing angle the reflected and transmitted rays are at right angles to each other.

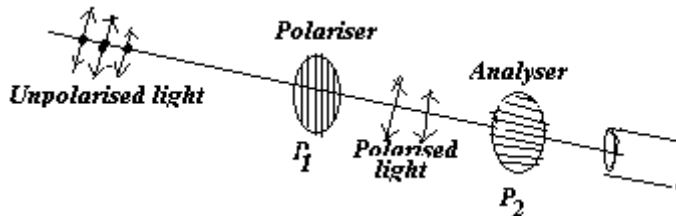
From Brewster's law $\mu = \tan p = \sin p / \cos p$ (1)

From Snell's law $\mu = \sin p / \sin r$ (2)

From Eqns (1) and (2), we get $\sin r = \cos p = \sin (90 - p)$
 or $r = 90 - p$ or $r + p = 90$

Therefore, the reflected and refracted rays are at right to each other. It should be noted that the refractive index of a substance varies with the wavelength of the incident light and hence the polarizing angle will be different for different wavelengths. Therefore for complete polarization the light should be monochromatic.

Law of Malus:



When unpolarized light falls on the polarizing sheet P₁, it will transmit only those wave train components whose electric vectors vibrate parallel to this direction and will absorb those that vibrate at right angles to this direction. The emerging light from P₁ will then be plane polarized, the intensity of which can be analysed by rotating P₂ (called analyser) using the law of Malus

According to Malus, when a completely plane polarized light beam is incident on the analyzer, the intensity of the polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the analyzer and the plane of polarizer

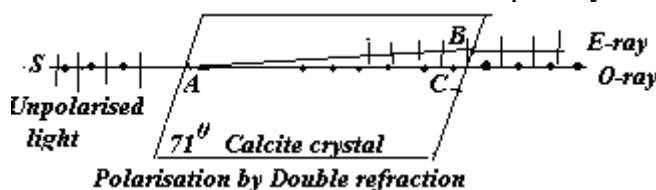
A second polarizing sheet (analyzer) P₂ is placed in the path of the emerging polarized light from P₁. P₂ is rotated about the direction of propagation. Let the amplitude of the plane polarized light falling on P₂ be E_m. Then the amplitude of light that emerges from P₂ is E_m cos θ where θ is the angle between the polarizing directions of P₁ and P₂ . Since the intensity of light is proportional to the square of the amplitude, the transmitted light intensity I varies with θ according to $I = I_m \cos^2\theta$ where I_m is the maximum transmitted intensity which occurs when the polarizing directions of P₁ and P₂ are parallel i.e., when θ = 0 or 180⁰. When θ = 90⁰ i.e., when the two planes are perpendicular I = 0. These results are experimentally observed in case of two tourmaline crystals.

Consider a beam of unpolarized light is incident on a pile of glass plates. All light reflected out of the original ray is polarized. After passing through several successive reflecting surfaces, the intensity of the emerging reflected beam can be increased. Also the perpendicular components are progressively

removed from the transmitted beam, making it more completely polarized with the electric field vector parallel to the plane of incidence.

Double refraction :

The refractive index of a medium μ and the speed of light v in the medium are related by $\mu = (c/v)$ where c is the velocity of light in free space. There are certain optical media in which v and μ are independent of the direction of propagation of light and of its state of polarization. Such media are called "optically isotropic media". Liquids, amorphous solids such as glass and crystalline solids having cubic symmetry are examples of isotropic media. There are many crystalline solids such as quartz and calcite whose optical properties are direction dependent. Such substances are called anisotropic and the phenomenon of double refraction is related to such optically anisotropic media.



When a beam of unpolarized light is allowed to fall on a calcite or quartz crystal, it is split up into two refracted beams in place of the usual one as in glass. This phenomenon is called 'double refraction' or 'birefringence' and such crystals are called doubly refracting crystals. The two refracted beams are found to be plane polarized with their planes of vibration at right angles to each other. The vibrations of O-ray are perpendicular to the optic axis. The vibrations of the E-ray are along the optic axis. One of the refracted ray, represented by O-ray (ordinary ray) is found to obey Snell's law of refraction. The second ray, represented by extraordinary ray or E-ray does not obey the laws of refraction

The O-ray and E-ray separate out in a crystal, when light is incident at an angle to the optic axis. They travel in different directions with different velocities. When light is incident in a direction normal to the optic axis, the O-ray and E-ray propagate along the same direction but with different velocities. When light is incident parallel to the optic axis, the two rays travel along the same direction with the same velocity

The ordinary ray travels in the crystal with the same speed v_0 in all directions and hence its wavefront is spherical. In other words, the crystal has for this ray a single refractive index μ_0 . The extra-ordinary ray has different velocities in different directions, its speed is greater, with a maximum value in the direction perpendicular to the optic axis. Hence its wavefront is ellipsoid. In other words, the refractive index μ_e for this ray, defined as c/v varies with direction. μ_e is maximum along the optic axis and minimum in a direction perpendicular to the optic axis. The quantities μ_0 and μ_e are called the principle indices of refraction for the crystal

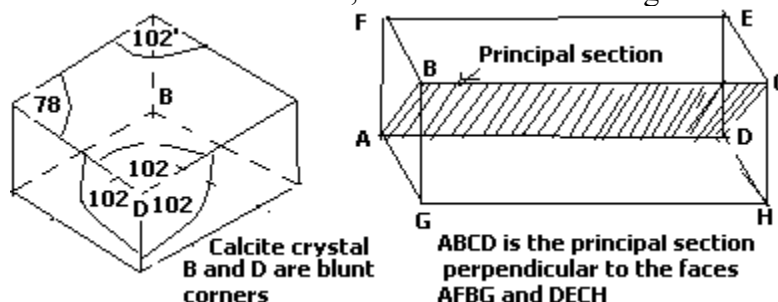
The spherical wavefront corresponding to O-ray and ellipsoid wavefront corresponding to E-ray touch each other at points which lie on the optic axis of the crystal *because the velocity of O-ray and E-ray is the same along the optic axis*

In certain crystals (calcite and tourmaline) called 'negative crystals', the ellipsoid lies outside the sphere. This shows that in negative crystals, the E-ray wavefront travels faster than O-ray wavefront except along optic axis, In certain crystals like ice and quartz called 'positive crystals', the sphere lies outside the ellipsoid. This shows that velocity of O-ray wavefront is greater than E-ray wavefront except along optic axis.

In certain crystals called 'uniaxial crystals' (calcite, tourmaline and quartz) there is only one direction (optic axis) along which the two refracted rays travel with the same velocity. In certain crystals, called *biaxial crystals* (topaz and mica), there are two such directions along which the velocities are the same

Calcite crystal:

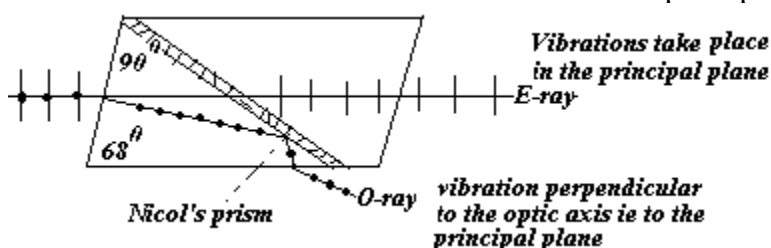
Calcite is a calcium carbonate (CaCO_3) crystal which belongs to the rhombohedral system. The six faces of rhombohedron are parallelograms each having angles of $101^{\circ}55'$ and $78^{\circ}5'$. There are two opposite corners A and B where all the three angles are obtuse ($101^{\circ}55'$). These corners are known as 'blunt corners'. At the other corners, there is one obtuse angle and two acute angles meet



A line passing through any one of the blunt corners and making equal angles with the three edges which meet at this corner is the direction of the "optic axis" of the crystal. The optic axis is a direction in a crystal parallel to the straight line through either of the blunt corners of the rhomb and makes equal angles with the three edges meeting there. The optic axis is a direction and not a particular line. Hence, an optic axis can be drawn through every point in the crystal. Any plane which contains the optic axis and is perpendicular to two opposite faces is called the "principal section" of the crystal.

Nicol's prism: Nicol's prism is an optical device made from calcite crystal (CaCO_3) and is used in many optical instruments to produce and analyse plane polarized light. Calcite being a double refracting crystal produces O-ray and E-ray when natural light passes through it. The O-ray is cut-off by total internal reflection leaving the E-ray alone.

Construction: Nicol's prism is constructed from a calcite crystal whose length is nearly three times its width. The two short end faces are cut down from 71° in the principal

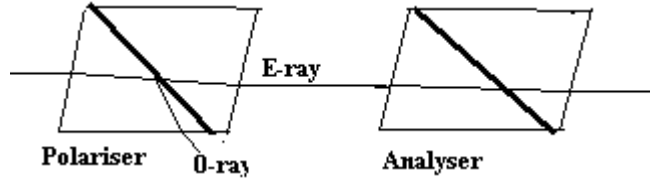


section to 68° to increase the field of view. The rhombo is then cut into two diagonally through one of the obtuse corners. The surfaces are polished and cemented together in their original position with Canada balsam which is a transparent glue with a refractive index ($\mu=1.55$) about midway between the refractive index of o-ray and that of e-ray. $\mu_e=1.486$ and $\mu_o=1.658$. The sides of the Nicol's prism are blackened to absorb any light coming out of the sides.

Action : When a beam of natural light SM enters one end of the faces in a direction parallel to the long side, it is doubly refracted and is split into ordinary plane polarized beam MP and extraordinary plane polarized beam ME. Since the natural angle of the end faces has been slightly changed, the ordinary ray is so refracted that it strikes the Canada balsam interface at an angle greater than the critical angle. As it is traveling from an optically denser medium to an optically rarer medium, it suffers total internal reflection and is deflected along the direction PK and is absorbed by the coat of black paint. The e-ray is not totally reflected because it is traveling from a rarer to a denser medium and is thus transmitted with no appreciable loss in intensity and comes out of the prism parallel to the original direction. So a single beam of plane polarized light is obtained from a Nicol prism. If the angle of incidence is greater than 14° , the e-ray will also suffer total internal reflection with the result that no light will emerge out of the prism.

Use of Nicol prism : The arrangement shown here consists of two Nicol prisms placed one after another in the path of incident light. The first Nicol prism produces plane polarized light, called 'polariser' and

the second Nicol prism is used to examine the state of the polarization of the transmitted light and is called the ‘analyser’.



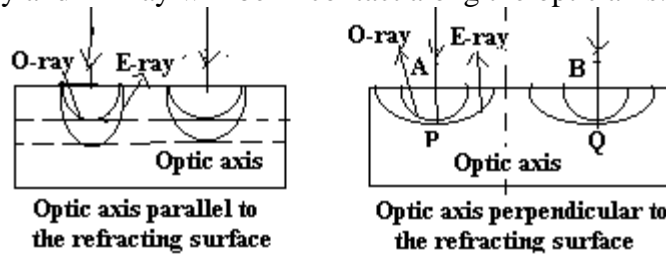
When the two Nicol prisms are placed with their principal sections parallel to each other, then the e-ray is transmitted by the first Nicol is freely transmitted by the second Nicol prism. On rotating one of the Nicol prisms, the intensity of the transmitted light decreases till it is reduced to zero when the principal sections of the two Nicol prisms are perpendicular to each other. It is so because the E-ray from the first Nicol prism forms an O-ray for the second and is therefore totally reflected.

If I_0 be the intensity of the transmitted beam when the principal sections of the two Nicol prisms are parallel and I_θ is the intensity when the principal sections are inclined at an angle θ , then according to the cosine law of Malus

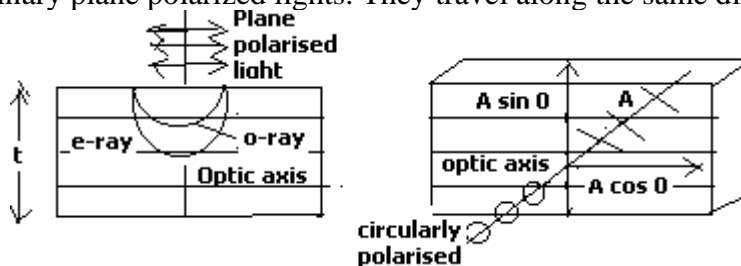
$$I_\theta = I_0 \cos^2 \theta$$

Propagation of light in doubly refracting crystals:

Optic axis parallel to the refracting face of the crystal: The secondary wavelets are generated at points the points of incidence A and B on the crystal surface. No double refraction occurs because both O-and E-waves propagate in the same direction but with different speeds as the wavefronts are separated and a progressively increasing phase difference arises between them. The spherical and elliptical wavefront for the O-ray and E – ray will be in contact along the optic axis.



Quarter wave plate: It is a thin plate of doubly refracting uniaxial crystal cut with its optic axis parallel to the surface. When plane polarized light falls normally on the surface, it is split up into ordinary and extraordinary plane polarized lights. They travel along the same direction but with different speeds.



Calcite being a negative crystal, the velocity of e-ray is greater than the velocity of o-ray ($\mu_e < \mu_o$). As a result a phase difference or a path difference is introduced between them. If ‘t’ is the thickness of the crystal plate, μ_o and μ_e are the refractive indices of the crystal with respect to o-ray and e-rays, then the equivalent air thickness of the plate is $\mu_o t$ and $\mu_e t$. The path difference between the two $\mu_o t - \mu_e t = t (\mu_o - \mu_e)$. **If the thickness of the crystal plate ‘t’ is such that it produces a phase difference of $\pi/2$ radians or a path difference of $\lambda/4$ between the O-ray and E-ray in passing through it, then it is called a quarter wave plate for that particular light of wavelength λ**

$$t (\mu_o - \mu_e) = \lambda/4 \text{ or } t = \lambda/4 (\mu_o - \mu_e) .$$

For a quartz crystal which is a positive crystal $t = \lambda/4 (\mu_e - \mu_o)$

A quarter wave plate is used for the production and detection of circularly polarized light. In combination with Nicol prism, it is used for analyzing any kind of polarized light. If the plane polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis is incident on a quarter wave plate, the emergent light is circularly polarized.

Half-wave plate:

It is a plate made from doubly refracting uniaxial crystal with its refracting faces cut parallel to the optic axis. **The thickness t of the plate is such that it introduces a path difference of $\lambda/2$ between the O-ray and the E-ray in passing through it when light is incident normally on the face of the crystal, then it is called a half-wave plate**

$$t (\mu_o - \mu_e) = \lambda/2 \quad \text{or} \quad t = \lambda/2 (\mu_o - \mu_e)$$

Production of polarized light:

Light exhibits three types of polarization: i) plane polarization ii) circular polarization and iii) elliptical polarization

Plane polarized light: When unpolarised light is incident on a Nicol prism, the emergent light from the Nicol prism is plane polarized. When Nicol prisms are used to produce plane polarized light, they are called *polarizers*

Mathematical treatment of plane, circular and elliptically polarized light:

Consider a beam of plane polarized light obtained from a Nicol prism falling normally on a calcite crystal with its optic axis cut parallel to the refracting faces. The plane polarized light on entering the crystal is split into E-ray and O-ray. The E-ray and O-ray traveling along the same direction with E-ray traveling faster than O-ray. After traveling through the crystal of thickness t , they may emerge with a phase difference δ between them, δ is a function of the crystal thickness t . Let A be the amplitude of the plane polarized light from the Nicol prism. The amplitude of the O-ray is $A \sin \theta$ (vibration perpendicular to the optic axis) and that of the E ray is $A \cos \theta$ (vibrations along the optic axis). θ is the angle between the plane of vibration and the direction of the optic axis.

The equations for E-ray and O-rays are

$$\text{E-ray} \quad x = A \cos \theta \sin (\omega t + \delta)$$

$$\text{O-ray} \quad y = A \sin \theta \sin \omega t$$

Let $A \cos \theta = a$; $A \sin \theta = b$; then $x = a (\sin \omega t + \delta)$ or $x/a = (\sin \omega t + \delta)$ and

$$y = b \sin \omega t; \quad \cos \omega t = \sqrt{1 - \sin^2 \omega t} = \sqrt{1 - y^2 / b^2}$$

Therefore $x/a = \sin \omega t \cos \delta + \cos \omega t \sin \delta = (y/b) \cos \delta + \sqrt{1 - y^2 / b^2} \sin \delta$

$$\text{or} \quad x/a - (y/b) \cos \delta = \sqrt{1 - y^2 / b^2} \sin \delta$$

Squaring both sides $x^2/a^2 + (y^2/b^2) \cos^2 \delta - (2xy/ab) \cos \delta = (1 - y^2/b^2) \sin^2 \delta$

$$x^2/a^2 + (y^2/b^2) \cos^2 \delta + (y^2/b^2) \sin^2 \delta - (2xy/ab) \cos \delta = \sin^2 \delta$$

$$x^2/a^2 + (y^2/b^2) - (2xy/ab) \cos \delta = \sin^2 \delta \dots \dots \dots (1)$$

This is the general equation of an ellipse

Case i: When $\delta = 0$, $\sin \delta = 0$ and $\cos \delta = 1$

The above equation becomes $x^2/a^2 + (y^2/b^2) - (2xy/ab) = 0$ or $(x/a - y/b)^2 = 0$

$$\text{or} \quad x/a - y/b = 0 \quad \text{or} \quad y = bx/a.$$

This is the equation of a straight line. Hence *the light will be plane polarized* with vibrations in the same plane as the incident light

Case ii: When $\delta = \pi/2, 3\pi/2, 5\pi/2 \dots$, $\sin \delta = 1$ and $\cos \delta = 0$

Equation (1) becomes $x^2/a^2 + y^2/b^2 = 1$. This is the equation of an ellipse and hence light will be *elliptically polarized*.

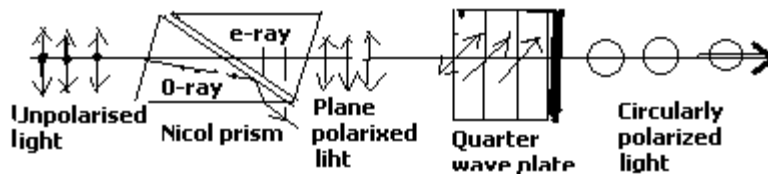
Case iii: $\delta = \pi/2$; $\sin \delta = 1$ and $\cos \delta = 0$ and $a = b$

Equation (1) becomes $x^2 + y^2 = a^2$. This is the equation of a circle of radius a . Hence the emerging light is *circularly polarized*, When $a = b$; $A \cos \theta = A \sin \theta$ or θ must be 45° . Hence the vibrations of the

incident plane polarized light on the crystal must make an angle of 45° with the direction of the optic axis to produce circularly polarized light

Production of Circularly polarized light:

Circularly polarized light is the result of two polarized light waves of equal amplitude vibrating at right angles to each other and having a phase difference of $\pi/2$. The result is a wave in which the E vector at each point has a constant magnitude but rotates around the direction of propagation



If the plane polarized light from a Nicol prism is incident normally on the face of a quarter wave plate such that its plane of vibration makes an angle of 45° with the direction of the optic axis of the quarter wave plate, then the light is split into extraordinary ray and ordinary ray of equal amplitudes. The quarter wave plate introduces a phase difference of $\pi/2$ between e- ray and o-ray. This results in the formation of a circular vibration and the outgoing light is circularly polarized. The amplitudes of the two component waves are equal i.e.,

$$A \cos 45 = A/\sqrt{2} = a \quad \text{and} \quad A \sin 45 = A/\sqrt{2} = a$$

At the time of emergence from the plate, the phase difference between the two waves is $\pi/2$. Hence their equations are

$$x = a \sin \omega t \quad \text{and} \quad y = a \sin (\omega t - \pi/2) = -a \cos \omega t$$

Now $\sin \omega t = x/a$; $\cos \omega t = -y/a$

Squaring and adding the above two terms, we get

$$\sin^2 \omega t + \cos^2 \omega t = x^2/a^2 + y^2/a^2 \quad \text{or} \quad x^2 + y^2 = a^2$$

This is the equation of a circle with radius 'a'. Hence the electric vectors of the wave executes circular motion.. In other words, the light emerging out of the quarter wave plate is circularly polarized.

Hence, circularly-polarized light is one in which the electric vectors execute circular vibrations having a constant time period and taking place in the transverse plane. Obviously, in this case, the amplitude of vibrations remains constant but its direction changes continually

Elliptically polarized light:

Elliptically polarized light is the result of two light waves of unequal amplitudes vibrating at right angles to each other and having a phase difference of $\pi/2$

Let us consider a beam of plane polarized light from a Nicol prism is incident normally on a quarter-wave plate such that the plane of vibration makes an angle θ with the optic axis. The incident wave is split up into two waves, one having vibrations parallel to the optic axis (E-ray) and the other vibrations perpendicular to the optic axis (O-ray). If A is the maximum amplitude of the incident plane polarized wave, then the maximum amplitude of the E-wave is $A \cos \theta$ and the O-wave is $A \sin \theta$. The two waves travel along the same path but with different speeds. When they emerge out of the quarter wave plate, they have a phase difference of $\pi/2$. Hence the equations just after they emerge out of the crystal are

$$x = A \cos \theta \sin \omega t \quad \text{and} \quad y = A \sin \theta \sin (\omega t - \pi/2)$$

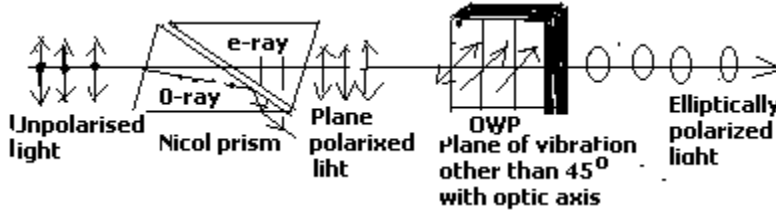
But $A \cos \theta = a$ and $A \sin \theta = b$

$$x = a \sin \omega t, \quad y = b \sin (\omega t - \pi/2) = -b \cos \omega t$$

Now $\sin \omega t = x/a$; $\cos \omega t = -y/b$

Squaring and adding $\sin^2 \omega t + \cos^2 \omega t = x^2/a^2 + y^2/b^2 = 1$

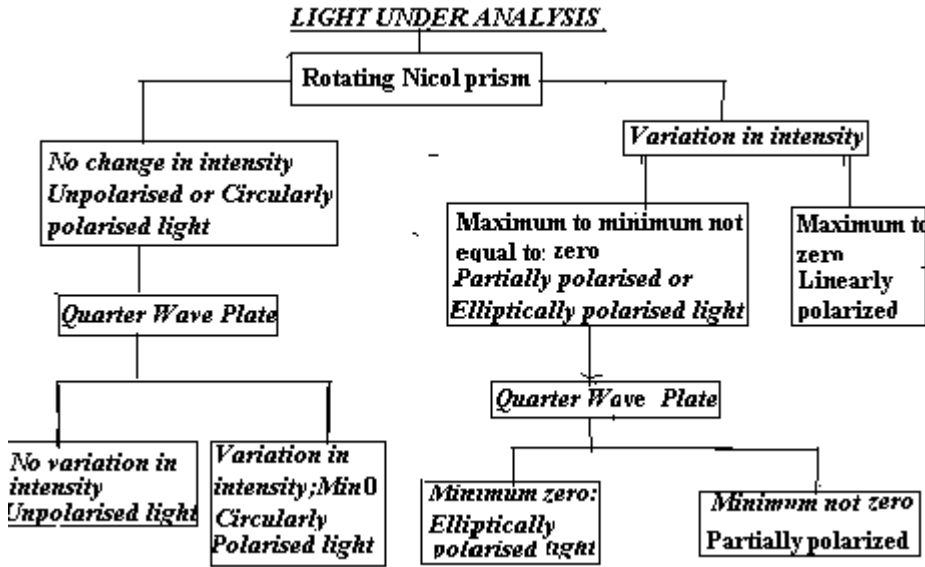
This is the equation of a symmetrical ellipse with major and minor axes of $2a$ and $2b$ respectively. Hence the light after coming out of the quarter wave plate becomes elliptically polarized



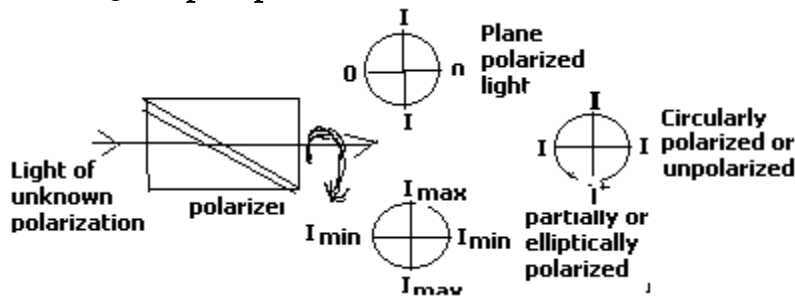
Elliptically polarized light is one in which the electric vectors execute elliptical vibrations, having a constant time period and taking place in a transverse plane i.e., in a plane perpendicular to the direction of propagation of light. The amplitude of vibration changes in magnitude as well as in direction

Analysis of the type of polarization of light:

In practice light may exhibit any one of the three types of polarization or may be unpolarized or partially polarized. The following steps are used in the analysis of the type of polarization.



i) A rotating Nicol prism (polarizer) is introduced into the path of light whose type of polarization is to be analysed. If the intensity of transmitted light varies between a maximum and zero, twice in one rotation then the light is *plane polarized*.

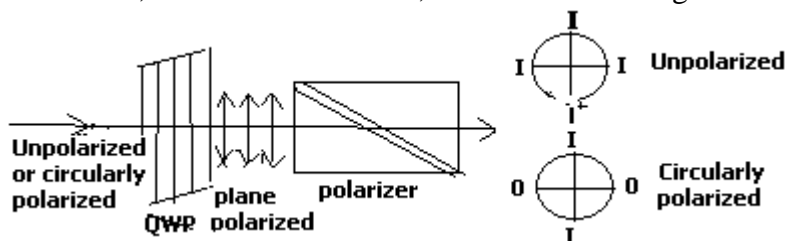


ii) On rotation of the polarizer if the intensity varies between a maximum and minimum and does not become zero, then the light is either *partially polarized or elliptically polarized*

iii) On rotation of the polarizer, if the intensity of transmitted light remains constant, then the incident light is either *circularly polarized or unpolarized*.

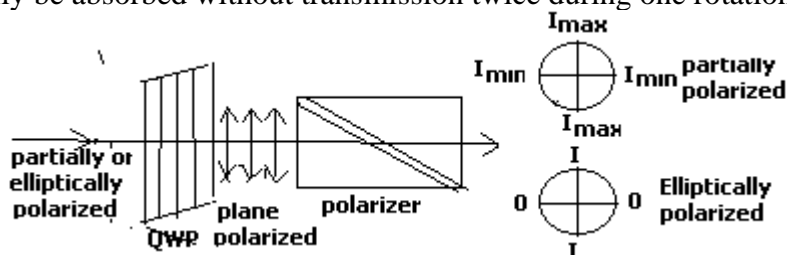
iv) To distinguish between the unpolarized light and circularly polarized light, a quarter wave plate is to be introduced before the polarizer. If the incident light is circularly polarized, the QWP converts it into

plane polarized light. On rotation of the polarizer, if the intensity of the transmitted light varies between a maximum and zero, twice in one rotation, then the incident light on the QWP is *circularly polarized*.



v) On rotation of the polarizer, if the intensity of the transmitted light stays constant, then the incident light on QWP is *unpolarized*.

vi) If the unknown light is elliptically polarized, it will become *plane polarized* after passing through the QWP and would totally be absorbed without transmission twice during one rotation of the polarizer.



vii) If the intensity of transmitted light from Nicol prism varies between a maximum and a minimum without becoming zero twice during one rotation of polarizer, then the light is *partially polarized*

Applications of polarization:

Polarizing plastic sheets or polaroids are used in the wind screens of automobiles to reduce the glare of head light lamps. Similarly, polaroid goggles are used to reduce the sun light glare. The size and shape of virus particles can be determined by polarization of ultraviolet light scattered from them. Very useful information about the structure of the atoms and nuclei is obtained from polarization studies of their emitted radiation. Liquid crystal display (LCD) display uses polaroid light to form letters and numbers in watches and calculators.

University Questions:

- 1) Write what is meant by i) plane polarized ii) elliptically polarized and iii) circularly polarized light . Briefly explain how these can be produced and detected
- 2) What is elliptically polarized light. How will you distinguish between elliptically polarized light and plane polarized light
- 3) Explain the construction and working of Nicol's prism
- 4a) Discuss an experimental setup to produce circularly polarized light from unpolarized light.
- b) Explain Quarter -wave plate and Half-wave plate
- 5(a) Explain double refraction and discuss the construction and working of Nicol prism. Explain how it can be used as polariser and analyzer
- (b) Explain circularly and elliptically polarized light
- 6(a) Explain the phenomenon of double refraction
- (b) Discuss the construction and working of Nicol prism
- (c) Calculate the thickness of a mica plate for making a quarter wave plate for light of wavelength 6000 AU. The indices of refraction for the ordinary and extra ordinary rays in mica are 1.586 and 1.592

Summary:

- 1) Light from a natural source such as an incandescent lamp is unpolarised

- 2) There are three types of polarization, namely i) plane polarization ii) circular polarization and iii) elliptical polarization
- 3) A mixture of plane polarized light and unpolarized light is known as partially polarized light
- 4) If the vibrations of the electric vector E in a light wave are parallel and confined to a single plane, such a light wave is said to be plane polarized or linearly polarized
- 5) If in a wave the electric vector does not confine to a single plane but goes on gradually rotating in space about the direction of propagation of wave as the wave advances and its magnitude remains constant, then the wave is said to be circularly polarized.
- 6) If in a wave advancing in space, the electric vector does not confine to a plane but goes on rotating around the direction of propagation of wave and at the same time undergoes change in magnitude, then the light is said to be elliptically polarized.
- 7) Any wave with any type of polarization can be regarded as a resultant of superposition of two plane polarized waves having been polarized in mutually perpendicular directions.
- 8) A plane polarized light can be produced from unpolarised light by reflection, refraction and double refraction.
- 9) An elliptically polarized light is the result of two plane polarized light waves of unequal amplitudes vibrating at right angles to each other and having a phase difference of $\pi/2$
- 10) A circularly polarized light is the result of two plane polarized light waves of equal amplitudes vibrating at right angles to each other and having a phase difference of $\pi/2$

- 11) If the thickness t of the calcite crystal is such that it produces a phase difference of $\pi/2$ radians or a path difference of $\lambda/4$ between O-ray and E-ray in passing through it, then it is called a 'quarter wave plate' for that particular light of wavelength λ . $t(\mu_o - \mu_e) = \lambda/4$ or $t = \lambda/4(\mu_o - \mu_e)$
- 12) If the thickness t of the calcite crystal is such that it produces a phase difference of π radians or a path difference of $\lambda/2$ between O-ray and E-ray in passing through it, then it is called a 'half-wave plate' for that particular light of wavelength λ . $t(\mu_o - \mu_e) = \lambda/2$ or $t = \lambda/2(\mu_o - \mu_e)$
- 13) If the plane polarized light whose plane of vibration is inclined at an angle of 45° to the optic axis is incident on a quarter wave plate, then the emergent light is circularly polarized
- 14) If the plane polarized light whose plane of vibration is inclined at an angle other than 45° to the optic axis is incident on a quarter wave plate, then the emergent light is elliptically polarized
- 15) When unpolarized light is incident on a Nicol prism, the emergent light from the Nicol's prism is plane polarized